## S3 - Ch. 4: Goodness of Fit and Contingency Tables

1. A researcher carried out a survey of three treatments for a fruit tree disease. The contingency table below shows the results of a survey of a random sample of 60 diseased trees.

|  | No action | Remove <br> diseased <br> branches | Spray with <br> chemicals |
| :--- | :---: | :---: | :---: |
| Tree died <br> within 1 year | 10 | 5 | 6 |
| Tree survived <br> for 1-4 years | 5 | 9 | 7 |
| Tree survived <br> beyond 4 years | 5 | 6 | 7 |

Test, at the $5 \%$ level of significance, whether or not there is any association between the treatment of the trees and their survival. State your hypotheses and conclusion clearly.
2. The number of times per day a computer fails and has to be restarted is recorded for 200 days. The results are summarised in the table.

| Number of restarts | Frequency |
| :---: | :---: |
| 0 | 99 |
| 1 | 65 |
| 2 | 22 |
| 3 | 12 |
| 4 | 2 |

Test whether or not a Poisson model is suitable to represent the number of restarts per day. Use a $5 \%$ level of significance and state your hypothesis clearly.
3. A research worker studying colour preference and the age of a random sample of 50 children obtained the results shown below.

| Age in years | Red | Blue | Totals |
| :---: | :---: | :---: | :---: |
| 4 | 12 | 6 | 18 |
| 8 | 10 | 7 | 17 |
| 12 | 6 | 9 | 15 |
| Totals | 28 | 22 | 50 |

Using a $5 \%$ significance level, carry out a test to decide whether or not there is an association between age and colour preference. State your hypotheses clearly.

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4. Five coins were tossed 100 times and the number of heads recorded. The results are shown in the table below.

| Number of <br> heads | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 6 | 18 | 29 | 34 | 10 | 3 |

(a) Suggest a suitable distribution to model the number of heads when five unbiased coins are tossed.
(b) Test, at the $10 \%$ level of significance, whether or not the five coins are unbiased. State your hypotheses clearly.
5. The Director of Studies at a large college believed that students' grades in Mathematics were independent of their grades in English. She examined the results of a random group of candidates who had studied both subjects and she recorded the number of candidates in each of the 6 categories shown.

|  | Maths grade <br> A or B | Maths grade <br> C or D | Maths grade <br> E or U |
| :--- | :---: | :---: | :---: |
| English grade <br> A or B | 25 | 25 | 10 |
| English grade <br> C to U | 15 | 30 | 15 |

(a) Stating your hypotheses clearly, test the Director's belief using a $10 \%$ level of significance. You must show each step of your working.

The Head of English suggested that the Director was losing accuracy by combining the English grades C to U in one row. He suggested that the Director should split the English grades into two rows, grades C or D and grades E or U as for Mathematics.
(b) State why this might lead to problems in performing the test.
6. A quality control manager regularly samples 20 items from a production line and records the number of defective items $x$. The results of 100 such samples are given in Table 1 below.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 or more |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 17 | 31 | 19 | 14 | 9 | 7 | 3 | 0 |

## Table 1

(a) Estimate the proportion of defective items from the production line.

The manager claimed that the number of defective items in a sample of 20 can be modelled by a binomial distribution. He used the answer in part (a) to calculate the expected frequencies given in Table 2.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 or more |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expected <br> frequency | 12.2 | 27.0 | $r$ | 19.0 | $s$ | 3.2 | 0.9 | 0.2 |

Table 2
(b) Find the value of $r$ and the value of $s$ giving your answers to 1 decimal place.
(c) Stating your hypotheses clearly, use a $5 \%$ level of significance to test the manager's claim.
(d) Explain what the analysis in part (c) tells the manager about the occurrence of defective items from this production line.

